E C O N O M I C S   B U L L E T I N

FIRM SIZE AND MONITORING

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Abstract

We present a model of optimal monitoring expenditures. For any technology that yields a conventional ``S−shaped" production function for monitoring, the optimal level of monitoring is shown to be higher in medium−sized firms than in both small and large firms. Further, the interaction between specialization and agency are shown to lead to an ``S−shaped" production function.

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FIRM SIZE AND MONITORING

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Abstract

We present a model of optimal monitoring expenditures. For any technology that yields a conventional "S-shaped" production function for monitoring, the optimal level of monitoring is shown to be higher in medium-sized firms than in both small and large firms. Further, the interaction between specialization and agency are shown to lead to an "S-shaped" production function.

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1 Introduction

Previous studies assume that larger firms are more difficult to monitor than firms of smaller size (Stigler [4], Garen [2]). As a result, large firms have to pay higher efficiency wages (Stigler [4] and Brown and Medoff [1])). However, if the production of monitoring is similar to the production of any other product, then the benefit of specialization may lead to easier monitoring in large firms. This paper explores this possibility and characterizes the relation between firm size and monitoring.

2 Firm Size and Monitoring

A firm has N employees, n of whom are allocated to production (hereafter referred to as workers) and the rest, $m = N - n$, are allocated to monitoring the employees in production (hereafter referred to as monitors). We refer to N as the *firm size* and m as the *size of the monitoring team.* Only one good is produced by the firm and the output U is determined by the monitoring level p and the number of the workers as $U = (N - m)u(p)$, where $u(p)$ is the output of each employee. The monitoring technology is such that $p = \tilde{p}(m, N) = \frac{G(m)}{N-m}$, where $G(m)$ is the amount of "monitoring," an intermediate product produced by the m monitors. The firm owner's optimization problem is thus:

$$max_m U = (N - m)u\Big(\frac{G(m)}{N - m}\Big). \tag{1}$$

Proposition 1 states sufficient conditions for larger firms to devote more resources to monitoring.

Proposition 1 *Let $m^*(N)$ be the optimal solution to optimization problem (1). Then, $\frac{dm^*}{dN} > 0$ if the monitoring technology and production technology are such that $D_1\tilde{p} > 0$, $u'(p) > 0$, $u''(p) < 0$, and the solution to Problem (1) is regular and interior.*[3].

Proof: The first-order-condition for the optimization problem $\max_m U(m; N)$ is

$$\tilde{J}(m; N) \equiv D_1 U$$

[3]The solution m^* is regular and interior if and only if $m^* \in (0, N)$ and $\frac{d^2U}{dm^2} < 0$.

$$\begin{aligned} &= -u + (N-m)u' \frac{(N-m)G'(m) + G(m)}{(N-m)^2} \\ &= -u + G'(m)u' + \frac{G(m)}{N-m}u' \ = \ 0. \end{aligned}$$

Differentiating both sides of the last equality with respect to N and solving for $\frac{dm^*}{dN}$ yields

$$\frac{dm^*}{dN} = \frac{u''(p)G(m)D_1\tilde{p}}{(N-m)D_1\tilde{J}(m;N)}.$$

The proposition then follows. ∎

Proposition 2 states that the relation between monitoring level and firm size is determined by the sign of $G''(m)$. Hence, any monitoring production function that is not increasing and concave over the whole domain provides a counter example to the conventional belief that large firms tend to have lower monitoring level. For instance, if $G(m)$ is 'S-shaped,' as usually assumed for production functions, then the relation between the optimal monitoring level and the firm size is non-monotonic.

Proposition 2 *Under the same assumptions as in Proposition 1, the relation between the optimal monitoring level p^* and the firm size N is completely determined by the sign of the second order derivative of the production function of monitoring, $G(m)$. Specifically, the sign of $\frac{dp^*(N)}{dN}$ is the same as that of $G''(m)$.*

Proof: Applying the results from Proposition 1,

$$\frac{dp^*}{dN} = D_1\tilde{p}\frac{dm^*}{dN} + D_2\tilde{p} = -\frac{\frac{u'(p)G(m)}{(N-m)^2 D_1\tilde{p}}G''(m)}{D_1\tilde{J}(m;N)}.$$

The proposition then follows from the second order condition of problem (1), namely that $D_1\tilde{J}(m;N) < 0$. ∎

3 "S-shaped" Production Function of Monitoring

This section shows how the interaction between specialization and agency induces an "S-shaped" production function for monitoring. Assume each worker has to accomplish tasks in the interval

[0,1]. Monitors are located in the "task-space" as follows: The ith monitor is located at $\frac{2i-1}{2m}, i = 1, 2, ..., m$, and supervises tasks in $\left[\frac{i-1}{m}, \frac{i}{m}\right]$. The monitors detect unsatisfactory performance in the interval of tasks. The probability that delinquency at task $\tilde{s}$ is detected by a monitor located at $\frac{1}{2m}$ is decreasing in the distance between the location of the monitor and the task, $s = |\tilde{s} - \frac{1}{2m}|$. Further, assume that the probability of detection equals to 1 when the distance is zero. Denote the probability of detecting delinquency during monitoring when the monitor exerts unity effort by $h(s)$; hence, $h'(s) < 0$, and $h(0) = 1$. The expected probability of any delinquency being caught by a monitor with unity effort level is, therefore, $H(m) = 2m \int_0^{\frac{1}{2m}} h(s)ds$. It can be shown that $H(m)$ is increasing in m as implied by the increasing returns in specialization.

The monitors themselves are monitored by the single firm owner. The probability a monitor who exerts an effort level lower than required is punished is thus $\frac{1}{m}$. The monitor's utility from exerting effort level of e is denoted $-f(e)$, where $f'(e) > 0$ and $f''(e) > 0$. The most severe penalty the firm can impose on the monitor is K, where $K > 0$ is exogenous and is expressed in terms of utility of the monitors. Denote by e^* the level of effort the monitor chooses to implement. The incentive-compatibility condition requires that $-f(e^*) \geq -\frac{1}{m}K - f(e)$, for all $e < e^*$. Hence, $e^* = e(m) = f^{-1}(\frac{K}{m})$. It can be shown that $e'(m) < 0$. In other words, the effort level chosen by the monitors is inversely related to the number of monitors due to the agency problem between monitors and the owner. Consequently, the expected probability of any delinquency being detected when monitored, ν, is given as $\nu = e(m)H(m) = f^{-1}(\frac{K}{m})H(m)$. The *monitoring level* (the probability of any delinquency being monitored AND detected), p, is then $p = \frac{m}{N-m}\nu$, where $\frac{m}{n} = \frac{m}{N-m}$ gives the probability by which a worker is monitored. The term $\frac{m}{N-m}$ is the *monitoring intensity*, while ν is the *monitoring effectiveness*, since it gives how effective each unit of monitoring intensity is in improving monitoring level.[4]

The production function generated in this model of monitoring is $G(m) = mH(m)e(m))$, where m reflects the complementarities between monitoring and production, $H(m)$ represents

[4] In previous models on monitoring, these concepts have not been distinguished from one another. For instance, Neal [3] uses the frequency of supervision as a proxy for the monitoring level, although it seems to be a proxy for monitoring intensity.

the benefit from specialization in monitoring, and $e(m)$ accounts for the agency costs. Under some fairly general conditions, the interaction between $H(m)$ and $e(m)$ provides an "S-shaped" production function $G(m)$, as usually assumed in economics.[5]

4 Discussion

Our results relating size and monitoring contrast with the conventional view in economics which does not consider the benefits from specialization in monitoring. Adam Smith viewed specialization as the most crucial contributing factor to productivity growth because finer specialization enables workers to develop skills within a narrower range and hence helps increase productivity. But the extent to which we can benefit from specialization is limited by the size and the scope of the market, according to Smith. In the model presented in this paper, a different factor limits the extent of specialization, where the benefit from specialization is constrained by the agency problem between monitors and firm owner.

[5]The specific conditions and the proof are available from the authors. As an example, consider the following technology: $h(s) = \frac{1}{(k_0 s+1)^2}$ for $s \geq 0$ and $f(e) = \frac{K_0 e}{K_0 - e}$ for $e \in [0, K_0]$, where $K_0 > 0$ is some physical constraint faced by the individual monitor, for instance, health condition, and $k_0 > 0$ gives information on how important specialization is in monitoring.

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